ATKINSON SCIENCE

THEORY GUIDE

International Standard Atmosphere Web Application

Keith Atkinson

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Atkinson Science welcomes your comments on this Theory Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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1 Introduction

You can find the ASL International Standard Atmosphere web application at the web address https://atkinsonscience.co.uk/WebApps/Aerospace/ISA.aspx. You will also find a user guide that you can download.

The International Standard Atmosphere is described in the document ISO 2533:1975 published in 1975 by the International Standards Organisation. The Standard is a model of the change in temperature and pressure with altitude in the Earth's atmosphere. ISO 2533:1975 presents tables of temperature and pressure and other properties of the atmosphere in terms of geometric altitude and geopotential altitude up to 80,000 m. The data up to 32,000 m are based on the US Standard Atmosphere 1962 and the ICAO Standard Atmosphere 1964. Data from recent research is used to extend the data to 80,000 m.

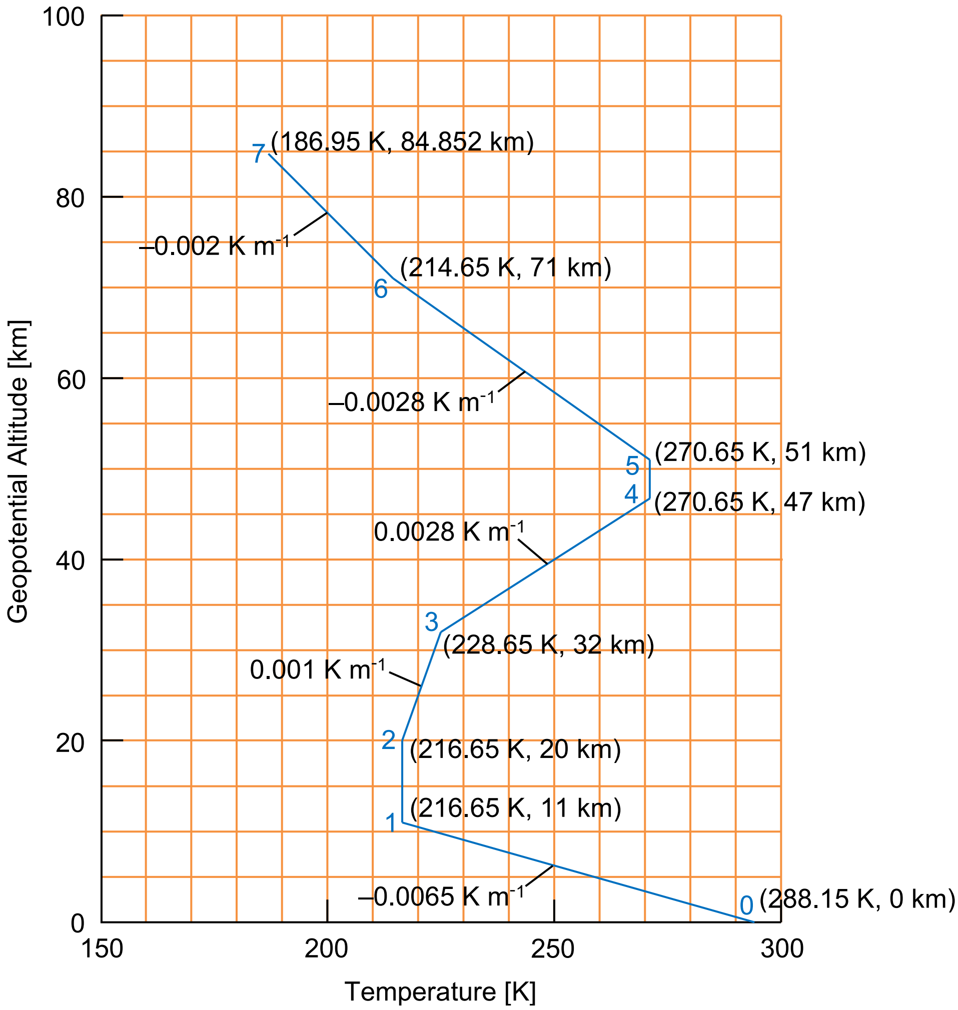
In ISO 2533:1975 the atmosphere is divided into layers over which the temperature is either constant or varies linearly with geopotential altitude, as shown in Figure 1. ISO 2533:1975 defines the temperature at sea level, the thickness of each layer and the *lapse rate* (K km^{-1}) of each layer. From this information the upper temperature of each layer can be calculated.

ISO 2533:1975 defines a number of constants and formulae by which the pressure and other properties of the atmosphere may be calculated from the temperature. The constants are set out in Table 1.

Table 1 Defined properties of the International Standard Atmosphere

Standard values at sea level	
Temperature T	288.15 K
Pressure p	101,325 Pa
Density ρ	1.2250 kg m^{-3}
Dynamic viscosity μ	1.7894 × 10^{-5} kg m^{-1} s^{-1}
Speed of sound c	340.29 m s^{-1}
Acceleration due to gravity g	9.80665 m s^{-2}
Other standard values	
Specific gas constant of air R_{Air}	287.05287 J kg^{-1} K^{-1}
Ratio of specific heats $\gamma = c_p/c_v$	1.4

Figure 1 Temperature of the International Standard Atmosphere



2 Geometric altitude and geopotential altitude

We define the *geometric altitude* z as the actual height above the Earth's surface. The acceleration due to gravity g varies with altitude and from elementary physics we can write:

$$\frac{g(z)}{g(0)} = \left(\frac{R_E}{R_E + z}\right)^2 \qquad (1)$$

where R_E is the radius of the Earth. The acceleration due to gravity at the Earth's surface g (0) is defined as 9.80665 m s^{-2} (see Table 1). R_E is defined as 6,356 km, which is the radius of the Earth along the line of latitude 45° north of the equator.

The equation for hydrostatic pressure is

$$dp = \ -\rho g(z) dz \qquad (2)$$

The variation in g with altitude makes this equation difficult to work with, so we introduce the *geopotential altitude* h such that

$$dp = -\rho g(0) dh \qquad (3)$$

Comparing Eqns. (2) and (3), we have

$$dh = \frac{g(z)}{g(0)} dz \qquad (4)$$

By substituting Eqn. (1) into Eqn. (4) we can eliminate the gravity terms:

$$dh = \left(\frac{R_E}{R_E + z}\right)^2 dz \qquad (5)$$

Integrating this equation gives

$$\int_0^h dh = \int_0^z \left(\frac{R_E}{R_E + z}\right)^2 dz \qquad (6)$$

Solving both sides of this equation gives

$$h(z) = \left(\frac{R_E}{R_E + z}\right) z \qquad (7)$$

Equation (7) enables us to determine the geopotential altitude h given the geometric altitude z.

By rearranging Eqn. (7) we can write the geometric altitude z in terms of the geopotential altitude h:

$$z(h) = \left(\frac{R_E}{R_E - h}\right) h \qquad (8)$$

3 Pressure and density for constant temperature

In this Section we will show how the atmospheric pressure and density are calculated in ISO 2533:1975 when the atmospheric temperature is constant.

The atmospheric air is assumed to be an ideal gas. Applying the equation of state for an ideal gas to the air gives

$$p = \rho R_{Air} T \qquad (9)$$

where R_{Air} is the specific gas constant of the air, which is 287.05287 J kg^{-1}K^{-1} (see Table 1).

Dividing Eqn. (3) by Eqn. (9) gives

$$\frac{dp}{p} = -\frac{\rho g(0) dh}{\rho R_{Air} T} = -\frac{g(0) dh}{R_{Air} T} \qquad (10)$$

Since T is a constant we can integrate Eqn. (10) immediately:

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{g(0)}{R_{Air} T} \int_{h_1}^{h_2} dh$$

or

$$\frac{p_2}{p_1} = e^{-\left(\frac{g(0)}{R_{Air} T}\right)(h_2 - h_1)} \qquad (11)$$

The equation of state (9) gives

$$\frac{\rho_2}{\rho_1} = \frac{p_2 RT}{p_1 RT} = \frac{p_2}{p_1} \qquad (12)$$

and substituting Eqn. (11) into Eqn. (12) gives

$$\frac{\rho_2}{\rho_1} = e^{-\left(\frac{g(0)}{R_{Air} T}\right)(h_2 - h_1)} \qquad (13)$$

Equations (11) and (13) give us the pressure change and the density change due to a change in the geopotential height $h_2 - h_1$.

4 Pressure and density for linearly varying temperature

In some of the atmospheric layers in Figure 1, the air temperature varies linearly with geopotential height. We define the *lapse rate* a as

$$a = \frac{dT}{dh} \qquad (14)$$

Since the change in temperature in the layer is linear, the lapse rate a must be a constant. Substituting Eqn. (14) into Eqn. (10) gives

$$\frac{dp}{p} = -\frac{g(0)}{aR_{Air}}\frac{dT}{T} \qquad (15)$$

Since a is a constant we can integrate this equation immediately:

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{-\left(\frac{g(0)}{aR_{Air}}\right)} \qquad (16)$$

Using the equation of state, we can write

$$\frac{\rho_2}{\rho_1} = \frac{p_2 R_{Air} T_1}{p_1 R_{Air} T_2} = \frac{p_2}{p_1}\left(\frac{T_2}{T_1}\right)^{-1} \qquad (17)$$

and substituting (16) into (17) gives

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{-\left(\frac{g(0)}{aR_{Air}}\right)}\left(\frac{T_2}{T_1}\right)^{-1}$$

or

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{-\left(\frac{g(0)}{aR_{Air}}+1\right)} \qquad (18)$$

Equations (16) and (18) give us the pressure change and the density change due to a change in the geopotential height $h_2 - h_1$ when the temperature varies linearly with geopotential height.

5 Standard atmospheric pressure and density variation

In ISO 2533:1975 the atmospheric pressure at sea level is defined as 101,325 Pa (1 atm.). There is a temperature gradient between points 0 and 1 in Figure 1, so we must use Eqn. (16) to calculate the pressure at point 1. We simply substitute the temperatures and lapse rate from Figure 1 and the sea level pressure into Eqn. (16):

$$p_1 = p_0\left(\frac{T_2}{T_1}\right)^{-\left(\frac{g(0)}{aR_{Air}}\right)} = 101325 \times \left(\frac{216.65}{288.15}\right)^{-\left(\frac{9.80665}{-0.0065\times 287.05287}\right)} = 22632\text{ Pa}$$

Between points 1 and 2 in Figure 1, the temperature is constant, so we must use Eqn. (11) to calculate the pressure at point 2. We substitute the geopotential altitudes and the constant temperature in Figure 1 and the pressure calculated for point 1 into Eqn. (11):

$$p_2 = p_1 e^{-\left(\frac{g(0)}{R_{Air}T}\right)(h_2-h_1)} = 22632 \times e^{-\left(\frac{9.80665}{287.05287\times 216.65}\right)(20000-11000)} = 5475\text{ Pa}$$

In this way, we can calculate the pressure at all the points 1 to 7 in Figure 1.

To calculate the density at points 1 to 7 we can use Eqn. (13) in place of Eqn. (11) and Eqn. (18) in place of Eqn. (16). Alternatively, we can calculate the density from the equation of state using the temperature and pressure at each point. Table 2 gives the calculated pressure and density at points 1 to 7.

Table 2 Pressure and density in the International Standard Atmosphere

Point	Geopotential altitude h [m]	Geometric altitude z [m]	Lapse rate a [K m^{-1}]	Temperature T [K]	Pressure p [Pa]	Density ρ [kg m^{-3}]
0	0	0	–0.0065	288.15	101,325	1.2250
1	11,000	11,109	0	216.65	22,632	0.3639
2	20,000	20,063	0.001	216.65	5,475	0.08804
3	32,000	32.162	0.0028	228.65	868.0	0.01322
4	47,000	47.350	0	270.65	110.9	0.001427
5	51,000	51,413	–0.0028	270.65	66.94	8.616×10^{-4}
6	71,000	71,802	–0.002	214.65	3.956	6.421×10^{-5}
7	84,852	86,000	NA	186.95	0.3734	6.958×10^{-6}

6 Dynamic viscosity and speed of sound

The dynamic viscosity of the air μ is calculated from Sutherland's law:

$$\mu = \mu(0)\left(\frac{T}{T(0)}\right)^{3/2}\left(\frac{T(0)+S}{T+S}\right) \qquad (19)$$

where the Sutherland constant S is 110 K. The sea-level temperature $T(0)$ and viscosity $\mu(0)$ are defined to be 288.15 K and 1.7894×10^{-5} kg m^{-1} s^{-1}, respectively (see Table 1).

The speed of sound c is given by

$$c = \sqrt{\gamma R_{Air} T} \qquad (20)$$

where the ratio of the specific heats $\gamma = c_p/c_v$ is defined to be constant and equal to 1.4 and the specific gas constant of the air R_{Air} is defined to be 287.05287 J kg^{-1}K^{-1} (see Table 1).

Table 3 gives the dynamic viscosity and speed of sound for points 0 to 7 in Figure 1.

Table 3 Dynamic viscosity and speed of sound

Point	Geopotential altitude h [m]	Geometric altitude z [m]	Temperature T [K]	Dynamic viscosity μ [kg m^{-1} s^{-1}]	Speed of sound c [m s^{-1}]
0	0	0	288.15	1.7894×10^{-5}	340.29
1	11,000	11,109	216.65	1.4219×10^{-5}	295.07
2	20,000	20,063	216.65	1.4219×10^{-5}	295.07
3	32,000	32.162	228.65	1.4871×10^{-5}	303.13
4	47,000	47.350	270.65	1.7038×10^{-5}	329.80
5	51,000	51,413	270.65	1.7038×10^{-5}	329.80
6	71,000	71,802	214.65	1.4109×10^{-5}	293.70
7	84,852	86,000	186.95	1.2538×10^{-5}	274.10